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Editorial

We start this pioneering work, which do not seek perfection as much as aiming to provide a scientific window that opens a wide area for all the distinctive pens, both in the University of Sabratha or in other universities and research centers. This emerging scientific journal seeks to be a strong link to publish and disseminate the contributions of researchers and specialists in the fields of applied science from the results of their scientific research, to find their way to every interested reader, to share ideas, and to refine the hidden scientific talent, which is rich in educational institutions. No wonder that science is found only to be disseminated, to be heard, to be understood clearly in every time and place, and to extend the benefits of its applications to all, which is the main role of the University and its scholars and specialists. In this regard, the idea of issuing this scientific journal was the publication of the results of scientific research in the fields of applied science from medicine, engineering and basic sciences, and to be another building block of Sabratha University, which is distinguished among its peers from the old universities.

As the first issue of this journal, which is marked by the Journal of Applied Science, the editorial board considered it to be distinguished in content, format, text and appearance, in a manner worthy of all the level of its distinguished authors and readers.

In conclusion, we would like to thank all those who contributed to bring out this effort to the public. Those who lit a candle in the way of science which is paved by humans since the dawn of creation with their ambitions, sacrifices and struggle in order to reach the truth transmitted by God in the universe. Hence, no other means for the humankind to reach any goals except through research, inquiry, reasoning and comparison.

Editorial Committee

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Invitation

The Editorial Committee invites all researchers "Lectures, Students, Engineers at Industrial Fields" to submit their research work to be published in the Journal. The main fields targeted by the Journal are:

- Basic Science.
- Medical Science & Technology.
- Engineering.

Refereeing

The Editorial Committee delivers researches to two specialized referees, in case of different opinions of arbitrators the research will be delivered to a third referee.

Editorial Committee

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SMOOTHING EFFECTS FOR A MODEL OF QUASI GEOSTROPHIC EQUATION

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Abstract

In this paper, we prove an estimation on Lebesgue space $L^p(\mathbb{R}^2), p \in [2, \infty[$, for a model of quasi geostrophic equation. For this, we use the Lagrange coordinates, and passing the L^p estimation for the equation localized.

Keywords: mechanics fluids; Incompressible fluid flow; Lebesgue spaces.

Introduction

We are interesting in this research, to a model of the quasi-geostrophic equation of the form

$$\begin{cases} \boldsymbol{\partial_t \theta + u \cdot \nabla \theta + \left(|D|^{\frac{1}{2}} + I\right) \theta = f} \\ \boldsymbol{div\, u = 0,} \\ \boldsymbol{\theta|_{t=0} = \theta_0,} \end{cases} \qquad \boldsymbol{(1.1)}$$

where θ is the scalar function represents the potential temperature and the $2D$ velocity field $u = u(x,t) = (u^1, u^2)$, $x \in \mathbb{R}^2$, $t \in \mathbb{R}_+$, is determined by Riesz transform $R_i, \forall i = 1,2$ of θ, that is

$$\boldsymbol{u = (-R_2\theta\, ,\, R_1\theta),}$$

Where $R_2\theta = \partial_2 |D|^{-1}\theta, and\ R_1\theta = \partial_1 |D|^{-1}\theta$. The fractional differential operator $|D| = (-\Delta)^{\frac{1}{2}}$ is defined by its Fourier transform

$$\boldsymbol{\mathcal{F}(|D|u) = |\xi| \mathcal{F}(u).}$$

The differential operator $u \cdot \nabla$ is defined respectively by

$$\boldsymbol{u.\nabla = \textstyle\sum_{i=1}^{2} u_i \partial_i.}$$

and the operator $div\, u$ is defined by

$$\boldsymbol{div\, u = \sum_{i=1}^{2} \partial_i u^i\ .}$$

The first equation of (1.1) serves as a $2D$ models arising in geophysical fluid dynamic (Pedlosky, J., 1987) and the second equation $div\ v = 0$, describe the incompressibity of the fluid. This equation has been intensively investigated and much attention is carried to the problem of global well-posedness. Some previous studies are established by numerous authors and in different functional spaces see for example (Chae, D. and Lee, J., 2004), (Constantin, P. and Wu, J., 2001), (Constantin, P. and Wu, J., 1999), and (Hmidi, T. et al., 2001).

The main goal of this work is to establish a smoothing effect for a model of quasi geostrophic equation in Lebesgue space, $L^p(\mathbb{R}^2), p \in [2, \infty[$.

The paper is organized as follows. In section 2, we give some definitions and recall some well-known results that will be need in the next section. In section 3, results and discussion are shown, and some conclusions are drawn in section 4.

Basic Concepts

In this section, we recall some notations and some functional spaces as a Lebesgue space L^p. We give also some results used in the paper.

- We denote by c any positive constant than will change from line to line.
- For any A and B, we say that $A \lesssim B$, if there exist a constant $C > 0$ such that $A \leq CB$.ss

Definition 2.1

We define the usual Lebesgue space $L^p(\mathbb{R}^d), p \in [1, +\infty[$, by the space of all function f such that,

$$\|\boldsymbol{f}\|_{\boldsymbol{L^p}} := \left(\int |\boldsymbol{f}(\boldsymbol{x})|^{\boldsymbol{p}} \boldsymbol{dx}\right)^{\frac{1}{p}} < \infty,$$

and for $p = \infty$, we say that $f \in L^\infty$, if

$$\|\boldsymbol{f}\|_{\boldsymbol{L^\infty}} := \boldsymbol{sup_x}|\boldsymbol{f}(\boldsymbol{x})| < \infty.$$

We define the dyadic decomposition of the full space $\mathbb{R}^2$ and recall the Littlewood-Paley operators, see for example (Bahouri, H., et al., 2011) and (Chemin, J-Y., 1998). There exist two nonnegative radial functions $\chi \in \mathcal{D}(\mathbb{R}^2)$ and $\varphi \in \mathcal{D}(\mathbb{R}^2/\{0\})$ such that,

$$\chi(\xi) + \sum_{q \geq 0} \varphi(2^{-q}\xi) = 1, \quad \forall \xi \in \mathbb{R}^2,$$

$$\sum_{q\in\mathbb{Z}} \boldsymbol{\varphi}(\,\mathbf{2}^{-q}\boldsymbol{\xi}) = \mathbf{1}, \forall \boldsymbol{\xi} \in \mathbb{R}^{2}/\{\mathbf{0}\},$$

$$|\boldsymbol{p}-\boldsymbol{q}| \geq \mathbf{2} \Rightarrow \boldsymbol{supp}\ \boldsymbol{\varphi}(\,\mathbf{2}^{-p}.) \cap\ \boldsymbol{upp}\ \boldsymbol{\varphi}(\,\mathbf{2}^{-q}.) = \boldsymbol{\phi},$$

$$\boldsymbol{q} \geq \mathbf{1} \Rightarrow \boldsymbol{supp}\ \boldsymbol{\chi} \cap\ \boldsymbol{upp}\ \boldsymbol{\varphi}(\,\mathbf{2}^{-q}.) = \boldsymbol{\phi}.$$

Let $h = \mathcal{F}^{-1}\varphi$ and $\bar{h} = \mathcal{F}^{-1}\chi$, the frequency localization operators Δ_q and S_q are defined by:

$$\Delta_{\boldsymbol{q}}\boldsymbol{f} = \boldsymbol{\varphi}(\mathbf{2}^{-q}\boldsymbol{D})\boldsymbol{f}, \qquad \boldsymbol{S_q f} = \boldsymbol{\chi}(\mathbf{2}^{-q}\boldsymbol{D})\boldsymbol{f}$$

$$\Delta_{-\mathbf{1}}\boldsymbol{f} = \boldsymbol{S_0 f}, \qquad \Delta_{\boldsymbol{q}}\boldsymbol{f} = \mathbf{0} \quad \boldsymbol{for\ q} \leq -\mathbf{2}.$$

We will need to the following inequality

Lemma 2.1 (Holder inequality)

If (f,g) belongs to $L^p \times L^q$ for any $(p,q,r) \in [1,\infty]^3$, and such that

$\frac{1}{r} = \frac{1}{p} + \frac{1}{q}$, then fg belongs to L^r and satisfies

$$\|\boldsymbol{fg}\|_{L^r} \leq \|\boldsymbol{f}\|_{L^p}\|\boldsymbol{g}\|_{L^q}.$$

The following result is needed in the proof of our main result see (Bahouri, H., et al., 2011) and (Sulaiman, S. A., 2010) for a proof.

Proposition 2.1

Let $p > 1$, and f in the Schwartz space $\mathcal{S}(\mathbb{R}^2)$. Then we have for every $q \in \mathbb{N}$ that,

$$\boldsymbol{c}\ \mathbf{2}^{\frac{q}{2}}\ \|\Delta_{\boldsymbol{q}}\boldsymbol{f}\|_{L^p}^{p} \leq \int \left(|\boldsymbol{D}|^{\mathbf{1/2}}\Delta_{\boldsymbol{q}}\boldsymbol{f}\right)\left|\Delta_{\boldsymbol{q}}\boldsymbol{f}\right|^{\boldsymbol{p}-\mathbf{1}}\boldsymbol{sign}\ \Delta_{\boldsymbol{q}}\boldsymbol{f}\ \boldsymbol{dx}.$$

The following can be found in (Cordobq Q, Cordobq D., 2004) and (Hmidi, T., 2005).

Lemma 2.2

Let u be a smooth divergence free vector field. Let also f be a smooth function and θ is a smooth solution of (1.1). Then for every $p \in [1,\infty]$ we have

$$\|\boldsymbol{\theta}(\boldsymbol{t})\|_{L^p} \leq \|\boldsymbol{\theta_0}\|_{L^p} + \int_0^t \|\boldsymbol{f}(\boldsymbol{\tau})\|_{L^p} \boldsymbol{d\tau}\,.$$

The proof of the following result can be found in (Sulaiman, S. A., 2010).

Lemma 2.3

Let θ any smooth scalar function , and u be a divergence free vector field of $\mathbb{R}^2$ such that $\nabla u \in L^p$.Then we have for all $p \in [1, \infty]$,

$$\left\|\left[\Delta_q, \boldsymbol{u}.\boldsymbol{\nabla}\right]\boldsymbol{\theta}\right\|_{L^p} \lesssim \|\boldsymbol{\nabla u}\|_{L^p}\|\boldsymbol{\theta}\|_{L^\infty}.$$

Main Results

In this section, we give the definition of the vorticity in dimension two and we prove the main result of the paper.

Definition 3.1

Let u be a smooth divergence free vector field of $\mathbb{R}^2$. We define the vorticity ω of the vector u in two dimensional space by

$$\begin{aligned} \boldsymbol{\omega} &= \boldsymbol{curl\, u} \\ &= \boldsymbol{\nabla}.\boldsymbol{u} \\ &= \boldsymbol{\partial_1 u^2} - \boldsymbol{\partial_2 u^1}. \end{aligned}$$

The main result of the paper is the following

Theorem 3.1

Let u be a smooth divergence free vector field of $\mathbb{R}^2$ with vorticity ω and θ be a smooth solution of (1.1). Then for every $p \in [2, \infty[$, there exists a positive constant c, and such that for every $q \in \mathbb{N}$ and $t \in \mathbb{R}_+$,

$$\boldsymbol{2^{\frac{q}{2}}}\left\|\Delta_q\boldsymbol{\theta}\right\|_{L_t^1L^p} \leq \boldsymbol{c}\left(\left\|\Delta_q\boldsymbol{\theta_0}\right\|_{L^p} + \|\boldsymbol{\theta_0}\|_{L^\infty}\|\boldsymbol{\omega}\|_{L_t^1L^p} + \|\boldsymbol{f}\|_{L_t^1L^p}\right).$$

Proof

The idea of the proof will be done in the spirit of (Hmidi; T., 2005), (Pedlosky, J., 1978), and (Sulaiman, S. A. and Ebreed, J., 2023).

We localize in frequency the first equation of (1.1), and rewriting the equation in Lagrangian coordinates.

Let $q \in \mathbb{N}$, then the Fourier localized function $\Delta_q\theta$ satisfies

$$\Delta_q(\partial_t\theta) + \Delta_q(u.\nabla\theta) + \Delta_q\left(|D|^{\frac{1}{2}} + I\right)\theta = \Delta_q f$$

Using the notation $[\,\Delta_q\,, u.\nabla\,]\theta = \Delta_q(u.\nabla\theta) - u.\nabla\Delta_q\theta$, We get

$$\Delta_q(u.\nabla\theta) = [\,\Delta_q\,, u.\nabla\,]\theta + u.\nabla\,\Delta_q\theta$$

This gives that

$$\partial_t\Delta_q\theta + u.\nabla\,\Delta_q\theta + [\,\Delta_q\,, u.\nabla\,]\theta + \left(|D|^{\frac{1}{2}} + I\right)\,\Delta_q\theta = \Delta_q\,f.$$

Therefore,

$$\partial_t\Delta_q\theta + u.\nabla\,\Delta_q\theta + \left(|D|^{\frac{1}{2}} + I\right)\,\Delta_q\theta = \Delta_q\,f - [\,\Delta_q\,, u.\nabla\,]\theta := F_q$$

Multiplying the above equation by $|\Delta_q\theta|^{p-2}\Delta_q\theta$, integrating by parts and using Holder inequality Lemma 2.1, we get

$$\begin{aligned}\frac{1}{p}\frac{d}{dt}\|\Delta_q\theta(t)\|_{L^p}^p + \int\left(|D|^{\frac{1}{2}}\Delta_q\theta\right)|\Delta_q\theta|^{p-2}\Delta_q\theta\,dx + \|\Delta_q\theta(t)\|_{L^p}^p \\ \le \|\Delta_q\theta(t)\|_{L^p}^{p-1}\|\Delta_q\,f\|_{L^p} + \|\Delta_q\theta(t)\|_{L^p}^{p-1}\|[\,\Delta_q\,, u.\nabla\,]\theta\|_{L^p}.\end{aligned}$$

Calculating the first term and using Proposition 2.1, we get

$$\begin{aligned}\|\Delta_q\theta(t)\|_{L^p}^{p-1}\frac{d}{dt}\|\Delta_q\theta(t)\|_{L^p} + \left(c\,2^{\frac{q}{2}} + 1\right)\|\Delta_q\theta\|_{L^p}^p \\ \le \|\Delta_q\theta(t)\|_{L^p}^{p-1}\|\Delta_q\,f\|_{L^p} + \|\Delta_q\theta(t)\|_{L^p}^{p-1}\|[\,\Delta_q\,, v.\nabla\,]\theta\|_{L^p}.\end{aligned}$$

Dividing by $\|\Delta_q\theta\|_{L^p}^{p-1}$ to obtain,

$$\frac{d}{dt}\|\Delta_q\theta(t)\|_{L^p} + \left(c\,2^{\frac{q}{2}} + 1\right)\|\Delta_q\theta(t)\|_{L^p} \le \|\Delta_q\,f\|_{L^p} + \|[\,\Delta_q\,, u.\nabla\,]\theta\|_{L^p}.$$

Multiplying the last inequality by $e^{c\left(t2^{\frac{q}{2}}+1\right)}$, we find

$$\frac{d}{dt}\left(e^{c\left(t2^{\frac{q}{2}}+1\right)}\|\Delta_q\theta(t)\|_{L^p}\right) \lesssim e^{c\left(t2^{\frac{q}{2}}+1\right)}\left(\|\Delta_q\,f\|_{L^p} + \|[\,\Delta_q\,, u.\nabla\,]\theta\|_{L^p}\right).$$

From Lemma 2.3, we have with $p > 1$, that

$$\frac{d}{dt}\left(e^{\left(ct2^{\frac{q}{2}}+1\right)}\left\|\Delta_q\theta(t)\right\|_{L^p}\right)\lesssim e^{c\left(t2^{\frac{q}{2}}+1\right)}(\|f\|_{L^p}+\|\nabla u\|_{L^p}\|\theta\|_{L^\infty})$$

$$\lesssim e^{c\left(t2^{\frac{q}{2}}+1\right)}(\|f\|_{L^p}+\|\omega\|_{L^p}\|\theta_0\|_{L^\infty}).$$

In the last line, we have used the following inequality see (Bahouri, H., et al., 2011), and (Chemin, T-Y.,1998),

$$\|\nabla u\|_{L^p}\lesssim\|\omega\|_{L^p},\qquad p>1.$$

Integrating the differential inequality we get,

$$\left\|\Delta_q\theta(t)\right\|_{L^p}\lesssim e^{-c\left(t2^{\frac{q}{2}}+1\right)}\left\|\Delta_q\theta_0\right\|_{L^p}+\int_0^t e^{-c(t-\tau)2^{\frac{q}{2}}}\|f(\tau)\|_{L^p}d\tau$$
$$+\|\theta_0\|_{L^\infty}\int_0^t e^{-c(t-\tau)2^{\frac{q}{2}}}\|\omega(\tau)\|_{L^p}d\tau.$$

Taking L_t^1 and using convolution inequalities, we obtain

$$2^{\frac{q}{2}}\left\|\Delta_q\theta\right\|_{L_t^1L^p}\leq c\left(\left\|\Delta_q\theta_0\right\|_{L^p}+\|\theta_0\|_{L^\infty}\|\omega\|_{L_t^1L^p}+\|f\|_{L_t^1L^p}\right),$$

which is the desired result.

Conclusion

We have proved an estimation on Lebesgue space $L^p(\mathbb{R}^2), p\in[2,\infty[$ for a model of quasi geostrophic equation by localizing the equation, and using the Lagrangian coordinates.

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