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Editorial

We start this pioneering work, which do not seek perfection as much as aiming to provide a scientific window that opens a wide area for all the distinctive pens, both in the University of Sabratha or in other universities and research centers. This emerging scientific journal seeks to be a strong link to publish and disseminate the contributions of researchers and specialists in the fields of applied science from the results of their scientific research, to find their way to every interested reader, to share ideas, and to refine the hidden scientific talent, which is rich in educational institutions. No wonder that science is found only to be disseminated, to be heard, to be understood clearly in every time and place, and to extend the benefits of its applications to all, which is the main role of the University and its scholars and specialists. In this regard, the idea of issuing this scientific journal was the publication of the results of scientific research in the fields of applied science from medicine, engineering and basic sciences, and to be another building block of Sabratha University, which is distinguished among its peers from the old universities.

As the first issue of this journal, which is marked by the Journal of Applied Science, the editorial board considered it to be distinguished in content, format, text and appearance, in a manner worthy of all the level of its distinguished authors and readers.

In conclusion, we would like to thank all those who contributed to bring out this effort to the public. Those who lit a candle in the way of science which is paved by humans since the dawn of creation with their ambitions, sacrifices and struggle in order to reach the truth transmitted by God in the universe. Hence, no other means for the humankind to reach any goals except through research, inquiry, reasoning and comparison.

Editorial Committee

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- Medical Science & Technology.
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Refereeing

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Editorial Committee

CONTENTS

COMPARING SOLVING LINEAR PROGRAMMING PROBLEMS WITH APPLICATIONS OF THE MOORE-PENROSE GENERALIZED INVERSE TO LINEAR SYSTEMS OF ALGEBRAIC EQUATIONS

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Abstract

In this paper, two types of systems were considered. These are the linear systems of algebraic equations (LSAE) as well as linear programming problems (LPP). These systems were studied utilizing the theory of the Moore-Penrose generalized inverse (MPGI) of matrices. We show that the TORA software using to solve linear programming problems. The MPGI of matrices was used to solve linear systems of algebraic equations by means of comparing this solution with solving linear programming in which the TORA software was employed.

Keywords: Linear Algebraic Systems; MPGI; Rank; Least Squares Solution; Linear Programming Problem; TORA software.

Introduction

Linear programming is considered one of the important topics in operations research, as it is concerned with finding competitive and rapid solutions to problems (Akpan, N. and Iwok, I.A, 2016). In addition, it is also considered to be one of the most important mathematical ways for improving what is available in ways

There are several studies that highlighted the role of linear programming in decision-making including the one of (Zaineb Alkawash and Asmaa Omar Mubayrash, 2023).

The linear programming problem is one which optimizes (maximizes or minimizes) a linear function subject to a finite collection of linear constraints (B Z Argawi, B. Z, 2018).

In 1947, Georg. B. Dantzig formulated the general LPP and devised the Simplex Method for solving these LPP (Francois Ndayiragije, 2017).

The system of equation

$$\boldsymbol{Ax} = \boldsymbol{b}\,;\, \boldsymbol{A} \in \mathbb{C}^{\boldsymbol{m}\times\boldsymbol{n}}, \boldsymbol{x} \in \mathbb{C}^{\boldsymbol{n}}, \boldsymbol{b} \in \mathbb{C}^{\boldsymbol{m}} \qquad \textbf{(1)}$$

The concept of the Moore-Penrose generalized inverse of matrices has been explained in many references.

The MPGI will be used to solve the linear systems of algebraic equations $Ax = b$ with coefficients matrix A. A solution utilizing the MPGI is minimal least squares solution (Kanan, A.M., Elbeleze, A.A., & Abubaker, A., 2019).

If $\in \mathbb{C}^{m\times n}$, then $A^{\dagger} \in \mathbb{C}^{m\times n}$ is unique, and it called the Moore-Penrose generalized inverse of A (Kanan, A., 2021).

In 2008, Kyrchei gaven Cramer's rule for quaternion systems of linear equations. In 2015, he introduced the Cramer's rule for some generalized inverse and obtained the least squares solution with the minimum norm for (1) when $rank(A) = n$ and when $rank(A) = r \le m < n$. In addition, he also obtained the least squares solution with the minimum norm for system $Ax = b$for some cases, and other results for other systems (Kanan, A., 2021).

Linear Programming Model

The general linear programming model with **n** decision variables and **m** constraints in the following form:

- **Objective function:**

  (Maximizes or minimizes) $\mathbf{Z} = \boldsymbol{c_1 x_1 + c_2 x_2 + \cdots + c_n x_n = \sum_{j=1}^{n} C_j\, X_j}$

- **Constraints:**

$$\begin{aligned} &\boldsymbol{a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n (\le, \ge, =) b_1} \\ &\boldsymbol{a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n (\le, \ge, =) b_2} \qquad \textbf{(2)} \\ &\qquad\qquad\qquad \vdots \qquad\qquad\qquad\qquad \vdots \\ &\boldsymbol{a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n (\le, \ge, =) b_m} \end{aligned}$$

- **Non-negativity constraint:**

  $\boldsymbol{x_1, x_2, \dots, x_n \le 0}$ **.**

Formally, the above LPP model having **n** decision variables can be summarized in the following form

Maximize or Minimize $\boldsymbol{Z = \sum_{j=1}^{n} C_j X_j}$

subject to $\boldsymbol{\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} x_j} (\le, \ge, =)$; $(\boldsymbol{i = 1, 2, \dots, m})$

$$\boldsymbol{X_j \leq 0} \;\; ; (\boldsymbol{j = 1, 2, \dots, n})$$

where c_j, a_{ij}, b_i are constants (Akpan, N. and Iwok, I.A, 2016), (B Z Argawi, B.Z, 2018).

Definitions and Theorems

Let

$\boldsymbol{Max\ Z = CX}$

subject to

$\boldsymbol{AX = b}$ **(3)**

$\boldsymbol{X \geq 0}$

be a linear program.

$$\boldsymbol{A = [A^j, A^J]\ ,\ C = [C^j, C^J]\ \ ,\ X = \left[X_j, X_J\right]}$$

Where A^j is the matrix relating to the basic variables, A^J is the matrix relating to the non- basic variables, x_j are the basic variables and x_J is the non-basic variables, C^j the cost vector related to the base j and C^J the cost vector is not related to the base J (Francois Ndayiragije, 2017).

Suppose that the linear system of the algebraic equation

$$\boldsymbol{A^j x_j = b}\ ; \boldsymbol{A^j} \in \mathbb{C}^{\boldsymbol{m \times n}}, \boldsymbol{x_j} \in \mathbb{C}^{\boldsymbol{n}}, \boldsymbol{b} \in \mathbb{C}^{\boldsymbol{m}} \qquad \textbf{(4)}$$

Then the system has unique solution which is $x_j = (A^j)^{-1}b$ if coefficient matrix A^J is nonsingular matrix.

In case A is a singular matrix or rectangular matrix, we will use the Moore-Penrose generalized inverse for matrix A^j (MPGI) denoted by $(A^j)^{\dagger}$ to solve the system (4); $x_j = (A^j)^{\dagger}b$ (Kanan, A.M., Elbeleze, A.A., & Abubaker, A., 2019). There exist many formulas for MPGI of coefficient matrix A^j depending on theranks of A (full column rank, full row rank, and full factorization rank) (Kanan, A.M, Elbeleze, A.A, & Abubaker, A., 2019).

We will use one formula when coefficients matrix A^j is full factorization (A^j is not of full rank), by Generalized Cramer Rule using MPGI.

Definition 1: (The full rank factorization) (Kanan, A., 2021).

A matrix $A = BC \in \mathbb{C}^{m\times n}$ with $rank(A) = r$ is said to be full rank factorization if B and C^T *have* r *columns.*

Theorem 1: (Kanan, A., 2021).

If $A = BC \in \mathbb{C}^{m\times n}$ where $A \in \mathbb{C}^{m\times n}$, $B \in \mathbb{C}^{m\times r}$, $C \in \mathbb{C}^{r\times n}$ and $r = rank\,(A) = rank(B) = rank(C)$, then $A^{\dagger} = C^*(CC^*)^{-1}(B^*B)^{-1}B^*$.

Definition 2: (Least squares solutions) (Kanan, A.M., Elbeleze, A.A., & Abubaker, A., 2019).

Suppose that $A \in \mathbb{C}^{m\times n}$ and $B \in \mathbb{C}^{m}$. Then a vector $w \in \mathbb{C}^{n}$ is called least squares solution to (4) if $\|Aw - b\| \leq \|Av - b\|$ for all $v \in \mathbb{C}^{n}$.

Theorem 2: (Kanan, A.M., Elbeleze, A.A., & Abubaker, A., 2019).

Suppose that $A \in \mathbb{C}^{m\times n}$ and $b \in \mathbb{C}^{m}$. Then $A^{\dagger}b$ is the minimal least solution to $Ax = b$. **Note** that, the system of equation (4) has minimal least squares solutions, $(A^{j})^{\dagger}b$ is called approximate solution to (4).

In the generalized Cramer Rule using MPGI, we are/were given solutions that are least squares solutions with the minimum norm, as the following theorem.

Theorem 3: (Kanan, A., 2021).

If $A = BC \in \mathbb{C}^{m\times n}$ with $rank(A) = rank(B) = rank(C) = r$, such that A is full rank factorization then least squares solutions with minimum norm for (4) is

$$\boldsymbol{x_j} = \frac{\sum_{i=1}^{r}\left|(CC^*)_i.a_j^*\right|\left|(B^*B)_i.\grave{b}_1^*\right|b_1+\sum_{i=1}^{r}\left|(CC^*)_i.a_j^*\right|\left|(B^*B)_i.\grave{b}_2^*\right|b_2+\cdots+\sum_{i=1}^{r}\left|(CC^*)_i.a_j^*\right|\left|(B^*B)_i.\grave{b}_m^*\right|b_m}{|CC^*||B^*B|};\ \boldsymbol{j} = \mathbf{1, 2, \ldots, n} \qquad \mathbf{(5)}$$

where, $B = (\grave{b}_{ij}) \in \mathbb{C}^{m\times r}$ $C = (c_{ij}) \in \mathbb{C}^{r\times n}$.

We set $\pi = C\left(A^{j}\right)^{\dagger}$ the linear program (3) becomes

$$\boldsymbol{Max\ Z = \pi b + \widehat{C}X}$$

subject to

$$\boldsymbol{\widehat{A}X = \widehat{b}}$$

$$\boldsymbol{X \geq 0}$$

Where $\hat{C} = C - \pi A$

$$\widehat{\boldsymbol{A}} = [\boldsymbol{I_m}, (\boldsymbol{A^j})^{\dagger}\boldsymbol{A^j}] \quad , \; \widehat{\boldsymbol{b}} = (\boldsymbol{A^j})^{\dagger}\boldsymbol{b}$$

I_m is an identity matrix of order **m.**

The solution of linear system is

$\boldsymbol{x_j = 0}$

$\boldsymbol{x_j} = (\boldsymbol{A^j})^{\dagger}\boldsymbol{b}$

If $\hat{C} \leq 0$ then

$\boldsymbol{Max\ Z = \pi b}$

In the case of the linear program

$$\boldsymbol{Min\ Z = AX}$$

subject to

$$\boldsymbol{AX = b}$$

$$\boldsymbol{X \geq 0}$$

If $\hat{C} \geq 0$ then

$Min\ Z = \pi b$. (Francois Ndayiragije, 2017).

Results

Consider the following linear program

$$\textbf{Max}\ \boldsymbol{Z = 3x_1 + 3x_2 + 9x_3}$$

subject to

$$\boldsymbol{x_1 + x_2 + 2x_3 \leq 12}$$

$$\boldsymbol{2x_1 + 2x_2 + 4x_3 \leq 20}$$

$$\boldsymbol{x_1, x_2, x_3 \geq 0}$$

With TORA software, the solution is obtained after the iteration 2:

Figure (1): Input Grid Linear Programming.

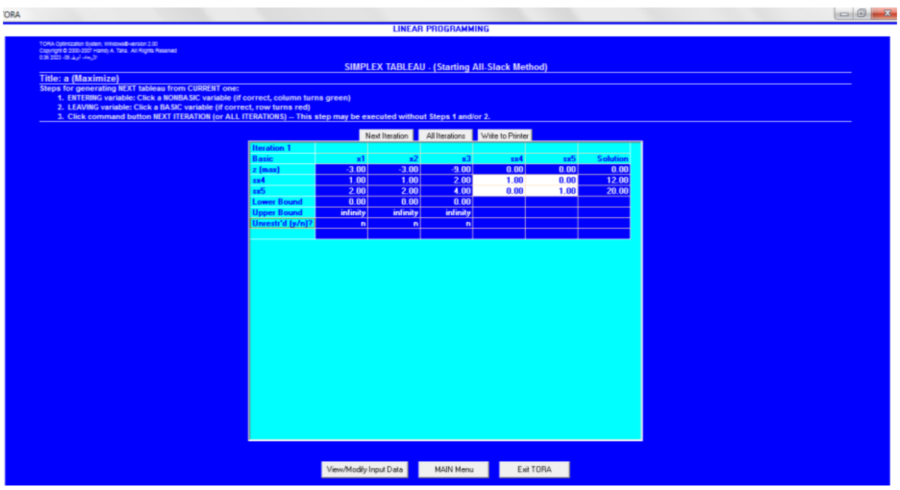

Figure (2): Iteration 1.

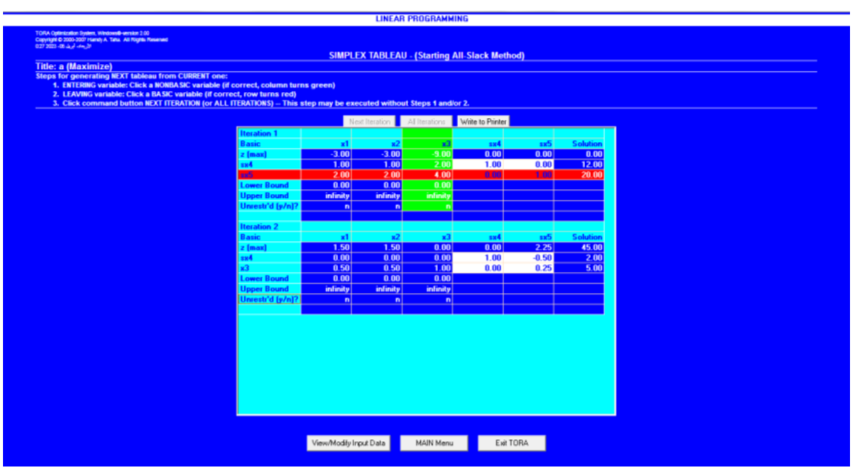

Figure (3): Iteration 1&2 (TORA, output screen).

Thus, the solution is:

Max Z=45

$\boldsymbol{x_1 = 0}$, $\boldsymbol{x_2 = 0}$, $\boldsymbol{x_3 = 5}$. Figure (3)

The above LPP under its standard form is:

$$\textbf{Max } \boldsymbol{Z = 3x_1 + 3x_2 + 9x_3}$$

subject to

$$\boldsymbol{x_1 + x_2 + 2x_3 \leq 12}$$

$$\boldsymbol{2x_1 + 2x_2 + 4x_3 \leq 20}$$

$$\boldsymbol{x_1, x_2, x_3 \geq 0}$$

From the LPP

$$A = \begin{pmatrix} 1 & 1 & 2 & 1 & 0 \\ 2 & 2 & 4 & 0 & 1 \end{pmatrix}$$

Now, we solve the linear system by generalized Cramer Rule MPGI to compute the value of.

$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_3}$.

By taking $A^j = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 2 & 4 \end{pmatrix}$

Rank $\left(A^j\right) = 1$, A^j is full rank factorization.

We can write A^j as

$A^j = BC$ with, $rank\left(A^j\right) = rank(B) = rank(C) = 1$

We are using Theorem1 to compute $A^\dagger$ have $B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $C = (1 \quad 1 \quad 2)$

$$B^* = (1 \quad 2) \Longrightarrow B^*B = 5 \Longrightarrow |B^*B| = 5 \Longrightarrow (B^*B)^{-1} = \frac{1}{5}$$

$$C^* = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \Longrightarrow CC^* = 6 \Longrightarrow |CC^*| = 6 \Longrightarrow (CC^*)^{-1} = \frac{1}{6}$$

$$\left(\mathrm{A}^{\mathrm{j}}\right)^{\dagger}=\mathrm{A}^{\dagger}=\mathrm{C}^{*}(\mathrm{CC}^{*})^{-1}(\mathrm{B}^{*}\mathrm{B})^{-1}\mathrm{B}^{*}=\begin{pmatrix}\frac{1}{30} & \frac{2}{30}\\ \frac{1}{30} & \frac{2}{30}\\ \frac{2}{30} & \frac{4}{30}\end{pmatrix}$$

Now, we use Theorem 3 to compute the value of $x_1, x_2,\ x_3$

$$x_1=\frac{|(CC^*)_1.a_1^*|\left|(B^*B)_1.b_1^{`*}\right|b_1+|(CC^*)_1.a_1^*|\left|(B^*B)_1.b_2^{`*}\right|b_2}{|CC^*||B^*B|}$$

$$\mathrm{x}_1=\frac{(1)(1)(12)+(1)(2)(20)}{(6)(5)}=1.7$$

$$x_2=\frac{|(CC^*)_1.a_2^*|\left|(B^*B)_1.b_1^{`*}\right|b_1+|(CC^*)_1.a_2^*|\left|(B^*B)_1.b_2^{`*}\right|b_2}{|CC^*||B^*B|}$$

$$\mathrm{x}_2=\frac{(1)(1)(12)+(1)(2)(20)}{(6)(5)}=1.7$$

$$x_3=\frac{|(CC^*)_1.a_3^*|\left|(B^*B)_1.b_1^{`*}\right|b_1+|(CC^*)_1.a_3^*|\left|(B^*B)_1.b_2^{`*}\right|b_2}{|CC^*||B^*B|}$$

$$\mathrm{x}_3=\frac{(2)(1)(12)+(2)(2)(20)}{(6)(5)}=3.47$$

$$\pi=C\left(A^j\right)^{\dagger}$$

$$= (3 \quad 3 \quad 9)\begin{pmatrix} \frac{1}{30} & \frac{2}{30} \\ \frac{1}{30} & \frac{2}{30} \\ \frac{2}{30} & \frac{4}{30} \end{pmatrix} = \begin{pmatrix} \frac{24}{30} & \frac{48}{30} \end{pmatrix}$$

$$\hat{C} = C - \pi A = (3 \quad 3 \quad 9 \quad 0 \quad 0) - \begin{pmatrix} \frac{24}{30} & \frac{48}{30} \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 & 1 & 0 \\ 2 & 2 & 4 & 0 & 1 \end{pmatrix}$$

$$= (-1 \quad -1 \quad 1 \quad -0.8 \quad -1.6) < 0$$

$$\text{Max Z} = \pi\text{b} = \begin{pmatrix} \frac{24}{30} & \frac{48}{30} \end{pmatrix}\begin{pmatrix} 12 \\ 20 \end{pmatrix} = 41.6$$

Note that, the solution of the linear system using the MPGI is often an approximate unique solution and is the least squares solution.

Interpretation of the Result

The above linear programming model was solved using the TORA program Figure (1), Figure (2), and Figure (3) which gave the optimal solution: $x_1 = 0\,, x_2 = 0\,, x_3 = 5$ and $z = 45$, solved system of algebraic equation by using generalized rule MPGI which gave the solution :$x_1 = 1.7\,, x_2 = 1.7\,, x_3 = 3.47$ and $z = 41.6$ because the solution of the linear systems using the MPGI is often an approximate unique solution and is least squares solution.

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