Journal of Applied Science

Biannual Peer Reviewed Journal Issued by Research and Consultation Center , Sabratha University

Issue (13)
September 2024

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Editorial

We start this pioneering work, which do not seek perfection as much as aiming to provide a scientific window that opens a wide area for all the distinctive pens, both in the University of Sabratha or in other universities and research centers. This emerging scientific journal seeks to be a strong link to publish and disseminate the contributions of researchers and specialists in the fields of applied science from the results of their scientific research, to find their way to every interested reader, to share ideas, and to refine the hidden scientific talent, which is rich in educational institutions. No wonder that science is found only to be disseminated, to be heard, to be understood clearly in every time and place, and to extend the benefits of its applications to all, which is the main role of the University and its scholars and specialists. In this regard, the idea of issuing this scientific journal was the publication of the results of scientific research in the fields of applied science from medicine, engineering and basic sciences, and to be another building block of Sabratha University, which is distinguished among its peers from the old universities.

As the first issue of this journal, which is marked by the Journal of Applied Science, the editorial board considered it to be distinguished in content, format, text and appearance, in a manner worthy of all the level of its distinguished authors and readers.

In conclusion, we would like to thank all those who contributed to bring out this effort to the public. Those who lit a candle in the way of science which is paved by humans since the dawn of creation with their ambitions, sacrifices and struggle in order to reach the truth transmitted by God in the universe. Hence, no other means for the humankind to reach any goals except through research, inquiry, reasoning and comparison.

Editorial Committee

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Journal Address:

Center for Research and Consultations, Sabratha University

Website: **https://jas.sabu.edu.ly/index.php/asjsu**

Email**:** **jas@sabu.edu.ly**

Local Registration No. (435/2018)

ISSN 💻 2708-7301

ISSN 🕮 2708-7298

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- Introduction.
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- Medical Science & Technology.
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Refereeing

The Editorial Committee delivers researches to two specialized referees, in case of different opinions of arbitrators the research will be delivered to a third referee.

Editorial Committee

CONTENTS

AFFECTION OF MOORE–PENROSE GENERALIZED INVERSE ON MATRICES OF CUBIC COMPLETE GRAPH AND NON-EMPTY REGULAR (COMPLETE) GRAPH

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Abstract

Background: The Moore-Penrose generalized inverse of a matrix was introduced by E. H. Moore (Adi Ben-Israel, 2003) as the "general reciprocal" of a matrix, and by Penrose (Ali Aziz Ali, 1983) as the "generalized inverse" of a matrix. This generalized inverse exists and is unique for every matrix (not necessarily square) with complex elements. The definitions employed by Moore - Penrose, although not identical, have been shown to be equivalent by Rado.

Methodology and aims: The aim of this paper is to discuss some new properties of a non- empty regular graph, and non- empty cubic complete regular graph and some properties of matrices of an incidence matrix $F(G)$ of G. Also, we use the affection of the Moore–Penrose Generalized Inverse (MPGI) of matrix $F(G)$ (Adi Ben-Israel, 2003, B. Noble, 1976, S. L. Campbell, C. D. Meyer, 1979, Li Weiguo, et al., 2013), we get to $F(G)(F(G))^{\dagger} = ((F(G))^{T})^{\dagger}(F(G))^{T}$, and we introduced some new theorems and results.

Results and Discussion: Let $G(V, E)$ be a non–empty cubic regular graph, then $(F(G))^{T}F(G) = S(G) + T(G) - I$. An element (i, j) of the matrix $(F(G))^{T}F(G)$ is the product of the two rows i, j of the matrix $(F(G))^{T}$. That is (i, j) in the matrix $(F(G))^{T}F(G)$ is $\deg(v_i) - 1$ in the graph, that is, $\deg(v_i) - 1 = r_{ii} \ni 1_{ii} \in I$. Additionally, If the number of vertices are even, then the resulting the graph are discrete (disjoint) graph such that number of the Composites equals to $\frac{n}{2}$, where $|V| = n$.

Conclusion: Our hope that this paper will motivate further study of affection MPGI of matrices of regular graphs when $p > 3$. Also on complete graphs when $p > 3$, and on all graphs in general.

Keywords: Graph theory; matrices theory; complete graph; regular graph; and Moore–Penrose generalized inverse.

Introduction

Consider the matrices of the graph, the matrices of the complete graph, and the matrices of the regular graph (Ali Aziz Ali, 1983, Berge C.,1962, Ramy Shahin, Sohil Mahfod, 2011, Wilson, R. J., 1972). Then the product of the transpose incidence matrix for the non-empty cubic complete (regular) graph (which is denoted by) with the incidence matrix of is equal to the summation of the degree matrix with the edge-adjacency matrix minus the identity matrix, which will be proved in theorem 2. And by using theorems 1, 2, we get theorems 3, and 4. Also, we studied the generalized affection of Moore-Penrose.

inverse of, and proved that a product of for the non-empty cubic complete (regular) graph with Moore-Penrose generalized inverse for that matrix (denoted by) is equal to Moore-Penrose generalized inverse of

$(F(G))^T$ (denoted by $((F(G))^T)^\dagger$) product $(F(G))^T$. Finally In theorem 5 we have proven the possibility of some results generalization (In case $p > 3$, where p is the degree of any vertex).

Preliminaries

Definition 1. (The Moore–Penrose generalized inverse (MPGI)) (S. L. Campbell, C. D. Meyer, 1979).

If $\in \mathbb{C}^{m\times n}$, then MPGI of A, denoted by $A^\dagger$, is the unique matrix in $\mathbb{C}^{n\times m}$ such that:

(MP1) $\mathbf{AA^\dagger A = A}$ **,**

(MP2) $\mathbf{A^\dagger AA^\dagger = A^\dagger}$ **,**

(MP3) $\mathbf{(AA^\dagger)^* = AA^\dagger}$ **,**

(MP4) $\mathbf{(A^\dagger A)^* = A^\dagger A}$ **.**

Where * denoted to transpose of matrix.

Note that, if the field is $\mathbb{R}$, then $*$ changes to the transpose.

Definition 2. A matrix $E \in \mathbb{C}^{m\times n}$ which has rank r is said to be in row echelon form if E is of the form:

$$\mathbf{E} = \left(\frac{\mathbf{C_{r\times n}}}{\mathbf{0_{(m-r)\times n}}}\right) ,$$

here the elements c_{ij} of ($C = C_{r\times n}$) satisfy the following conditions

(E1) $c_{ij} = 0$ when $> j$.

(E) The first non-zero entry in each row of C is 1.

(E_3) If $c_{ij}= 1$ is the first non-zero entry of the ith row. then the jth column of C is the unit vector $\mathbf{e}_i$ whose only non-zero entry is in the ith position.

Definition 3. A matrix $H \in \mathbb{C}^{n\times n}$ is said to be in hermite echelon form if its elements h_{ij} satisfy the following conditions:

(h_1) H is upper triangular (i.e. $h_{ij} = 0$ when $i > j$) .

(h_2) h_{ij} is either 0 or 1 .

(h_3) If $h_{ii} = 0$, then $h_{ik} = 0$ for every k , $1 \le k \le n$.

(h_4) If $h_{ii} = 1$, then $h_{ki} = 0$ for every $k \neq i$.

The next algorithm is applied for the square matrices .

Algorithm 1. (S. L. Campbell, C. D. Meyer, 1979)

To obtain the Moore – Penrose generalized inverse $A^{\dagger}$ for a square matrix $A \in \mathbb{C}^{n\times n}$

(I) Row reduce A^* to its hermite form H_{A^*} ,

(II) Select the distinguished columns of A^* . Label these columns $v_1, v_2, \dots, v_r$ and place them as columns in a matrix L .

(III) Form the matrix .

(IV) From $I - H_{A^*}$ and select the non-zero columns from this matrix . Label these columns $w_1, w_2, \dots, w_{n-r}$.

(V) place the columns of AL and the w_i's as columns in a matrix $M = (AL \vdots w_1 \vdots w_2 \vdots \cdots \vdots w_{n-r})$ and compute M^{-1} . (In fact, the only first rows r of M^{-1} are needed) .

(VI) Place the first r rows of M^{-1} (in the same order as they appear in M^{-1}) in a matrix called R .

(VII) Compute $A^{\dagger}$ as $A^{\dagger}=$.

In the following we introduce an algorithm to compute $A^{\dagger}$; $A \in \mathbb{C}^{m\times n}$.

Algorithm 2. (S. L. Campbell, C. D. Meyer, 1979)

To obtain (**MPGI**) of any $A \in \mathbb{C}^{m\times n}$.

(I) Reduce A to row echelon form E_A .

(II) Select the distinguished columns of A and place them as the columns in a matrix B in the same order as they appear in .

(III) Select the non-zero rows from E_A and place them as rows in a matrix C in the same order as they appear in E_A .

(IV) Compute $(C^*)^{-1}$ and $(B^*B)^{-1}$.

(V) $A^\dagger = C^*(CC^*)^{-1}(B^*B)^{-1}B^*$.

Definition 4. (Ali Aziz Ali, 1983, Eskander Ali, 2011, Hector Zenil, et al., 2014, Michael Wemyss, 2012/2013, Ramy Shahin, Sohil Mahfod, 2011, Wilson, R. J., 1972) A graph G is a non-empty finite set V of elements is called vertices with a set E of unordered pairs of vertices is called edges.

Definition 5. (Ali Aziz Ali, 1983, Eskander Ali, 2011, Hector Zenil, et al., 2014, Michael Wemyss, 2012/2013, Ramy Shahin, Sohil Mahfod, 2011, Wilson, R. J., 1972) The order of a graph G is the number of its vertices.

Definition 6. (Ali Aziz Ali, 1983, Ramy Shahin, Sohil Mahfod, 2011) Let $e = (u, v)$ be an edge between the vertices v , then e is called a loop if $u = v$.

Definition 7. A graph G is called a simple graph or a 1-graph if it's without loops .

Definition 8. The degree of any vertex v in a graph G is the number of edges which lay on it. The degree of any vertex v in a graph G denoted by $\deg(v_i)$

Definition 9. (Ali Aziz Ali, 1983) The graph G is called a complete graph if G is a simple graph and all different vertices are adjacented.

Definition 10. The graph G is called r-regular graph if $\deg(v) = r$ for all $v \in G$.

Note that:

1- Any complete graph is a regular graph.

2- The cubic graphs is a regular graph of third degree.

Definition 11. (Ramy Shahin, Sohil Mahfod, 2011) Let $G(V, E)$ be a graph with a set of vertices $V(G) = \{v_1, \dots, v_p\}$, and a set of edges $E(G) = \{e_1, \dots, e_q\}$. The adjacency matrix is a $p \times p$ matrix, $Z(G) = (z_{ij})$, where z_{ij} is the number of edges that joint between v_i and v_j.

Definition 12. (Ramy Shahin, Sohil Mahfod, 2011) Let $G(V, E)$ be a graph with a set of vertices $V(G) = \left\{v_1, \dots, v_p\right\}$ and a set of edges $(G) = \left\{e_1, \dots, e_q\right\}$. The edge - adjacency matrix is a $q \times q$ matrix $S(G) = \left[s_{ij}\right]$, (for some i, j and $s_{ij} = 1$ if e_i

adjacent to e_j, $s_{ij} = 0$ when e_i and e_j are not adjacent). Note that the edge adjacency matrix is symmetric and $s_{ii} = 0$ for all $i = 1, \dots, q$.

Definition 13. (Ramy Shahin, Sohil Mahfod, 2011) A matrix $F(G) = (f_{ij})$ where $f_{ij} = 1$ when the vertex v_i lies on the edge e_j , and $f_{ij} = 0$ when the vertex v_i does not lay on the edge e_j is called an incidence matrix. If the graph $G (V , E)$ does not contain loops, then the summation of elements of every column equals two. Also, the summation of elements of every row equals degree of vertex corresponding to that row.

Definition 14. (Ramy Shahin, Sohil Mahfod, 2011) The degree matrix is a $p \times p$ matrix $T(G) = (t_{ij})$, where $t_{ii} = \deg(v_i)$ for $i = 1 , 2 , \dots , p$ and $t_{ij} = 0$ when $i \neq j$.

Definition 15. Let $G (V , E)$ be a simple graph. The complementary graph denoted by $\overline{G}$ is a simple graph that its vertices are $V(G)$ and its edges are all (u, v) where (u, v) are not an edges in $G(V , E)$.

Definition 16. The graph g is called a sub graph of $G (V , E)$ if the set of vertices of g is subset of V, and every edges in g is an edge in G.

Definition 17. Let $G (V , E)$ be a graph. If V can be divided into two subsets V_1 and V_2 such that there is no edge exist joining vertices from V_1 to V_2 , then G called disconnected graph, and we called all subgraphs which was connected on vertex sets V_1 or V_2 by composite from G. Composites of G are the number of all sub graphs that are connected and disconnected between themselves.

Theorem 1. (Ramy Shahin, Sohil Mahfod, 2011)

If $G (V , E)$ is a non-empty graph, then

$$(\mathbf{F(G)})^{\mathbf{T}}\mathbf{F(G)} = \mathbf{Z(G)} + \mathbf{T(G)} \ .$$

3. The main results

Theorem 2. Let $G(V, E)$ be a non–empty cubic regular graph, then

$$(\mathbf{F(G)})^{\mathbf{T}}\mathbf{F(G)} = \mathbf{S(G)} + \mathbf{T(G)} - \mathbf{I} \ .$$

Proof :

An element (i , j) of the matrix $(F (G))^T F(G)$ is the product of the two rows i , j of the matrix $(F (G))^T$. That is (i , j) in the matrix $(F (G))^T F(G)$ is $\deg(v_i) - 1$ in the graph, That is , $\deg(v_i) - 1 = r_{ii} \ni 1_{ii} \in I$.

Let $i \neq j$. Then we have two cases :

(1) If $e_i , e_j \in E(G)$, then the element (i , j) <u>is</u> in the matrix $(F(G))^T F(G)$ equals to 1, because there is an adjacency between the edges e_i , e_j adjustment to 1, and it expresses by the number of vertices which joins between them.

(2) If $e_i , e_j \notin E(G)$, then the element (i , j) in $(F(G))^T F(G)$ equals to zero, because there is no adjacency between edges e_i , e_j.

Finally, we get

$$(\mathbf{F(G)})^{\mathbf{T}}\mathbf{F(G)} = \mathbf{S(G)} + \mathbf{T(G)} - \mathbf{I}.$$

Note that, if $F(G)$ in the size $p \times p$, and $F(G)$ is symmetric $(F(G) = (F(G))^T)$, then:

$$(\mathbf{F(G)})^{\mathbf{T}}\mathbf{F(G)} = \mathbf{S(G)} + \mathbf{T(G)} = \mathbf{Z(G)} + \mathbf{T(G)}\ .$$

Theorem 3. Let $G(V, E)$ be a non–empty graph **.** If G is a complete graph (regular graph) and $(F(G))^T F(G) = S(G) + T(G) - I$, then

(i) $\mathbf{deg}(\mathbf{v_i}) = \frac{\sum_{j=1}^{q} \mathbf{f_{ij}}}{2}$.

(ii) $\mathbf{deg}(\mathbf{v_j}) = \frac{\sum_{i=1}^{p} \mathbf{f_{ij}}}{2}$.

Proof:

$T(G)$ is the degree matrix, and it is the diagonal matrix. It's element t_{ii} analogous to the degree of the vertex i. Since $S(G)$ is the edge–adjacency matrix Then, the summation of entries of any row (column) equals the degree of the vertex analogous to the row(column) i plus 1. That means, $\deg(v_i) + 1$. Elements of row (column) from $T(G)$ with row (column) the analogous from $S(G)$, and by subtraction the row (column) from I the corresponding to that row (column) to be the summation is double degree of the corresponding vertex, division by 2 yields degree of vertex .

Theorem 4. Let $G(V, E)$ be a non–empty graph**,** and

$(F(G))^T F(G) = Z(G) + T(G)$. Then

(i) $\mathbf{deg}(\mathbf{v_i}) = \frac{\sum_{j=1}^{q} \mathbf{f_{ij}}}{2}$.

(ii) $\mathbf{deg}(\mathbf{v_j}) = \frac{\sum_{i=1}^{p} \mathbf{f_{ij}}}{2}$ **.**

Proof :

$T(G)$ is a matrix of the degree and the matrix $Z(G)$ is adjacency matrix

Thus, addition any row (column) equals to double degree of the vertex, division by 2 yields degree of the vertex.

Theorem 5. Let $G(V, E)$ be a non–empty regular graph then

$$\mathbf{det}(\mathbf{T}(\mathbf{G})) = (\mathbf{deg}(\mathbf{v_i}))^{\mathbf{n}}$$

Where $|V| = n$, and $\det(T(G))$ is the determinate of the matrix $(T(G))$.

Proof:

Since $(T(G))$ is a diagonal matrix. Then the product elements of diameter central equal to determinate $(T(G))$.

$$\left(\prod_{\mathbf{i=1}}^{\mathbf{p}} \mathbf{t_{ii}}\right)=\mathbf{det}(\mathbf{T}(\mathbf{G}))$$

On the other hand, $(T(G))$ is the degree matrix, that means t_{ii} is a degree of the vertex i. So G is a regular graph. Hence

$$\mathbf{det}(\mathbf{T}(\mathbf{G})) = (\mathbf{deg}(\mathbf{v_i}))^{\mathbf{n}}.$$

Corollary 1. $\deg(v_i) = \sqrt[n]{\det(T(G))}$.

Theorem 6. Let $G(V, E)$ be a non–empty regular completely cubic graph. Then

$$\mathbf{F}(\mathbf{G})(\mathbf{F}(\mathbf{G}))^{\dagger} = ((\mathbf{F}(\mathbf{G}))^{\mathbf{T}})^{\dagger}(\mathbf{F}(\mathbf{G}))^{\mathbf{T}} .$$

Proof : $((F(G))^{T})^{\dagger}(F(G))^{T}=((F(G))^{\dagger})^{T}(F(G))^{T}$ (from properties of **MPGI**)

$$= (\mathbf{F}(\mathbf{G})(\mathbf{F}(\mathbf{G}))^{\dagger})^{\mathbf{T}} = \mathbf{F}(\mathbf{G})(\mathbf{F}(\mathbf{G}))^{\dagger} .$$

Example 1. consider the following graph G,

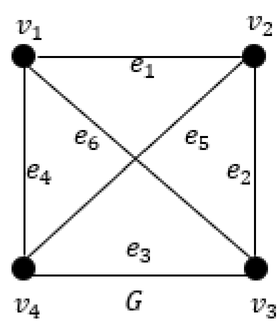

Figure (1)

where:

$$\mathbf{Z(G)} = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{bmatrix},$$

$$\mathbf{S(G)} = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\mathbf{F(G)} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{bmatrix},$$

$$\mathbf{T(G)} = \begin{bmatrix} \mathbf{3} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{3} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{3} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{3} \end{bmatrix}.$$

We find that

$$\mathbf{F(G)(F(G))^{\dagger}} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}.$$

Also,

$$((\mathbf{F(G)})^{\mathbf{T}})^{\dagger}((\mathbf{F(G)})^{\mathbf{T}} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}.$$

Corollary 2. If G a cubic complete graph Then $\mathrm{FF}^{\dagger} = \mathrm{I}($ (This means that

$$\mathbf{FF^{\dagger} = \frac{1}{3}T)) .}$$

Remark :

Note that, the last theorem is right too for the cubic regular graph, but the resultant was matrix with non-integer real entries for some cases. And not necessarily equals that I.

Results :

Let $\mathrm{G(V, E)}$ be a non–empty regular completely graph and

$\mathrm{F(G)(\ F(G)\ )^{\dagger} = ((\ F(G))^{T})^{\dagger}(\ F(G))^{T}}$ whereupon, then

(1) If the number of vertices are even, then the resulting the graph are discrete (disjoint) graph such that number of the Composites equals to $\frac{n}{2}$, where $|V| = n$.

Example 2. From example (1) the resulting graph will be denoted by J is as the Figure (2).

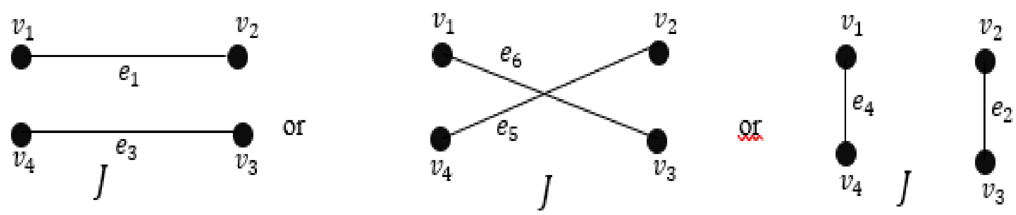

Figure (2)

(2) If (1) is actualize then the complementary graph $\overline{G}$ for the resultant graph from $F(G)(\,F(G)\,)^{\dagger} = ((\,F(G))^{T})^{\dagger}(\,F(G))^{T}$ is a regular graph .

Example 3. From example (1), the complementary graph $\overline{G}$ for the resultant graph J in example (2) is given in Figure (3).

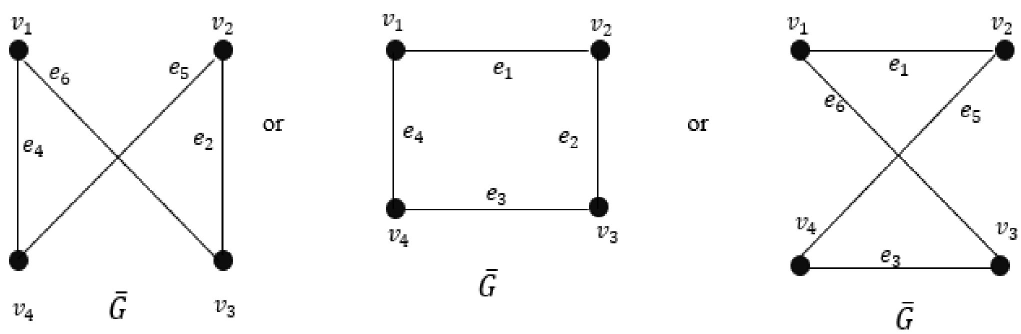

Figure (3)

(3) The complementary graph $\overline{G}$ in point (2) degree every vertices equal $(\deg(v_i) - 1)$, where $\deg(v_i)$ is the degree of vertex v_i in the graph G.

(4) The union of the complementary graph $\overline{G}$ and the resultant graph from $F(G)(\,F(G)\,)^{\dagger} = ((\,F(G))^{T})^{\dagger}(\,F(G))^{T}$ gives the graph G, that is

$$\mathbf{J \cup \overline{G} = G\ .}$$

(5) In previous points (from (1) to (4)) it is not necessarily that $\deg(v_i) = 3$.

Recommendations

We hope that this paper will be induced further study of effect MPGI of matrices of regular-graphs when $p > 3$. As well on complete-graphs when $p > 3$, also on all graphs in general.

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